

NOTATION

z, r, φ , cylindrical coordinate system; v_z, v_r, v_φ , components of the averaged velocity; v' , pulsation component of the velocity in the direction of the averaged velocity vector; v_0 , average-flow-rate velocity of flow; H , radial gap of the annular channel; y , radial distance from the inner wall; b , width of mixing layer; m , degree of concurrent flow; S , stream swirling; μ, ν , coefficients of viscosity. Indices: 0, average-flow-rate; 1, inner; 2, outer stream; m , maximum; t , turbulent.

LITERATURE CITED

1. G. N. Abramovich et al., Turbulent Mixing of Gas Jets [in Russian], Nauka, Moscow (1974).
2. A. A. Sviridenkov and V. V. Tret'yakov, "Experimental investigation of the mixing of turbulent, oppositely swirled jets in the initial section of an annular channel," *Inzh.-Fiz. Zh.*, 44, No. 2, 205-210 (1983).
3. G. N. Abramovich, The Theory of Turbulent Jets [in Russian], Fizmatgiz, Moscow (1960).
4. V. V. Tret'yakov and V. I. Yagodkin, "A calculated investigation of turbulent swirled flow in a pipe," *Inzh.-Fiz. Zh.*, 37, No. 2, 254-259 (1979).
5. I. Wygnanski and H. E. Fiedler, "The two-dimensional mixing region," *J. Fluid Mech.*, 41, Part 2, 327-361 (1970).
6. B. A. Zhestkov, Principles of the Theory and Calculation of the Thermal State of the Walls of Rocket Engine Combustion Chambers [in Russian], Ufim. Aviat. Inst., Ufa (1979).

CALCULATION OF THE CHARACTERISTICS OF A MIXING

CO₂ GASDYNAMIC LASER WITH A NOZZLE UNIT OF HONEYCOMB

CONSTRUCTION

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A mathematical model of a gasdynamic laser with parallel supersonic mixing of the components and its applications to the choice of the geometrical characteristics of the nozzle unit of honeycomb construction are presented.

One of the main problems in the creation of gasdynamic lasers (GDL) based on mixing is working out the construction of the nozzle apparatus simultaneously assuring the fullest realization of the energy stored in the exciting gas (N₂, CO), good optical quality of the medium, and sufficient pressure for exhausting into the atmosphere. From this point of view, the use of nozzle units of honeycomb construction, consisting of a large number of small-scale, axisymmetric nozzles having separate supply of the exciting and radiating gases with parallel supersonic mixing of the components has recently evoked great interest [1]. However, the available experimental results were obtained on specific constructions and evidently do not reflect the potentialities of this scheme. On the other hand, a unit of honeycomb construction, thanks to its exceptional simplicity and technological effectiveness, can be built with the most varied geometrical characteristics.

The composite gasdynamic pattern of formation of the supersonic stream of active medium, the interaction of the supersonic axisymmetric jets of the individual nozzles and the presence of wakes behind their rims, and the finite size of the mixing zones can have different and very significant influences on the characteristics of GDL. The use of the combustion products of various fuels, usually containing a number of admixtures besides nitrogen promoting the deactivation of the vibrationally excited molecules even before the interaction with the radiating gas, as the exciting gas requires great caution when using the assumption of "instantaneous mixing" to determine the GDL characteristics, especially from the aspect of their optimization. The model of "instantaneous mixing" was used in [2, 3] with certain additional assumptions for high values of the ratio of flow rates of the radiating and donor gases in [4]

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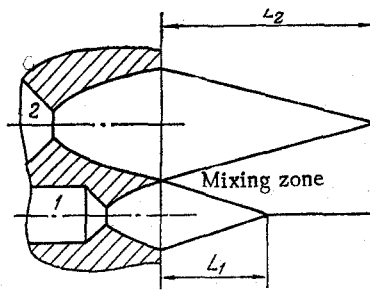


Fig. 1. Geometry of the mixing zone (plane analog):
1, 2) mixing streams.

and with the instantaneous introduction of losses in boundary layers and compression shocks in [1]. Allowance for the finite mixing rate for chemical lasers was made in [5, 6]. The model with instantaneous equalization of the stream with respect to the gasdynamic parameters and a finite rate of the relaxation processes was cited in [7] in application to mixing CO₂ GDL.

The actual geometrical characteristics of the unit and the influence of the boundary layers in the individual nozzles and the shock waves arising in the interaction of neighboring jets are taken into account in calculating model of a mixing GDL being offered. It is assumed that the influence of the jump in area from the nozzle cuts to the cross-sectional area of the gasdynamic channel takes place through adiabatic underexpansion (compression) of the streams. It is assumed that such a mechanism of initial interaction of the streams leads to pressure equalization in the cross section. The mixing is given by the mass rate of entrainment of gases into the mixing zone. Since the process of turbulent mixing is attainable in practice (even with an initially laminar character, the mixing zone is rapidly turbulized [8]), the rate of entrainment is given as a linear function of the ratio of the distance from the nozzle cut to the characteristic length L over which complete entrainment of the components into the mixing zone takes place [5]. The boundary layers formed in the individual nozzles flow into the gasdynamic channel in the form of "wakes," the "washing out" of which takes place over the same length L . Inequality of the lengths for the streams of exciting and radiating gases is presumed in the general case (Fig. 1). In [5] it was shown that for one set of lengths L_1 and L_2 it is possible to simultaneously reproduce correctly the experimental distributions of the amplification ratio in the stream and the pressure at the wall. It is also assumed that "instantaneous" mixing of the boundary layers from neighboring nozzles occurs beyond the cut of the nozzle unit and averaged wakes are present in the gasdynamic channel. A single system of equations is set up for the entire process of calculation from the critical cross sections of the individual nozzles to the end of the resonator. The general form of the equations is similar to the well-known solutions for GDL with preliminary mixing [9]:

$$\frac{dT}{dX} = - \left[(1-\lambda) \left(\sum \xi_i \theta_i \frac{de_i}{dX} + \frac{kI}{Pu} T \right) + T \left(\frac{1}{F} \frac{dF}{dX} - \frac{2}{1-\delta^*} \frac{d\delta^*}{dX} \right) + Q_r \right] / \left[(3.5 + 0.5\xi_{\text{H}_2\text{O}} - \xi_{\text{He}})(1-\lambda) - 1 \right], \quad (1)$$

$$\frac{du}{dX} = \frac{u\lambda}{(1-\lambda)} \left(\frac{1}{F} \frac{dF}{dX} - \frac{2}{1-\delta^*} \frac{d\delta^*}{dX} - \frac{1}{T} \frac{dT}{dX} + Q_r \right), \quad (2)$$

$$\frac{dP}{dX} = \begin{cases} -P/(\lambda u) du/dX & \text{for flow in a nozzle,} \\ f(X), & \end{cases} \quad (3)$$

$$\frac{de_i}{dX} = \left(\frac{de_i}{dX} \right)_{\text{rel}} + Q_{ei}, \quad (4)$$

where

$$\frac{1}{F} \frac{dF}{dX} = \begin{cases} 2/r dr/dX & \text{for flow in a nozzle,} \\ \frac{dG_1/dX \times \mu_2/\mu_1 \times u_2/u_1 \times T_1/T_2 + dG_2/dX}{G_1\mu_2/\mu_1 \times u_2/u_1 \times T_1/T_2 + G_2}, & \end{cases}$$

$$Q_r = -(1-\lambda)T(3.5 + 0.5\xi_{\text{H}_2\text{O}} - \xi_{\text{He}})/\mu \times d\mu/dX + (1-\lambda) \sum (\theta_i e_i \times d\xi_i/dX) - (1-\lambda) \sum \xi_i \theta_i e_i d\mu/dX/\mu + TQ_u - T(1-\lambda)/\lambda u^2 Q_h,$$

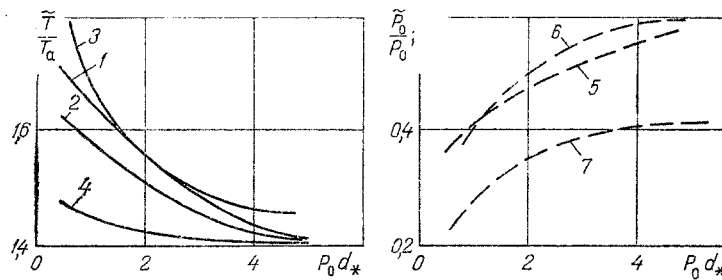


Fig. 2. Dependence of the reconstruction of the static temperature (solid lines) and the total pressure losses (dashed lines) on the binary similarity criterion (10^{-3} MPa·m): $A/A_* = 36$; $T_0 = 2700^\circ\text{K}$; $\bar{T}_w = 0.3$; 1, 7) profiled nozzle, $\theta_a = 0^\circ$; 2, 5) 6° ; 3, 5) 10° ; 4, 6) conical nozzle, $\theta_a = 10^\circ$.

$$Q_r = 1/\mu \, d\mu/dX - 1/G \, dG/dX + Q_u/\lambda,$$

$$Q_m = 1/G \{ (M_1 - M) dG_1/dX + (M_2 - M) dG_2/dX \},$$

$$M \equiv h, u, \xi_i \mu_{2,1} (G = G_1 \mu_2 + G_2 \mu_1), e_i \xi_{i,1,2} \mu_{2,1} (G = \xi_{i1} G_1 \mu_2 + \xi_{i2} G_2 \mu_2), \quad G = G_1 + G_2, \lambda = \mu u^2 / RT,$$

$$h = (3,5 + 0,5 \xi_{\text{H}_2\text{O}} - \xi_{\text{H}_2}) RT / \mu + R / \mu \sum \xi_i \theta_i e_i + u^2 / 2,$$

$(de_i/dX)_{\text{rel}}$ are the relaxation equations [9].

For flow in the nozzles $Q_t = Q_u = Q_{ei} = 0$, $G_1 = G_{10}X/L_1$, $G_2 = G_{20}X/L_2$, and $1/\mu d\mu/dX = 1/\mu \sum \mu_i d\xi_i/dX$.

The function f of Eq. (3) must be determined in the presence of a longitudinal pressure gradient (with widening of the walls of the gasdynamic channel, for example). The displacement thickness δ^* and the other integral characteristics of the boundary layer at the nozzle cut are determined by the well-known methods [10, 11]. The calculation is carried out in two steps for each nozzle separately (for the exciting and radiating gases) and from the nozzle cuts for the mixing zone. The initial conditions for calculating the flows in the nozzles are analogous to those of [9] and those for the flow in the mixing zone are analogous to those of [6]. The results obtained in the first step are the initial conditions for the solution in the mixing zone. Corrections connected with the additional expansion of the individual jets in the gasdynamic channel and with the interaction of the jets with each other are introduced "instantaneously" at the cut of the nozzle unit. The parameters at a tangential discontinuity after a shock interaction are found from the interference equation for two supersonic streams [12].

The values of the local (in the mixing zone) and average amplification ratios downstream from the cuts of the nozzles are calculated in the gasdynamic channel. The radiation output power is calculated for a Fabry-Perot resonator in the constant-intensity approximation.

The choice of the geometric characteristics of an individual nozzle is determined by the need to provide the maximum level of freezing-in of vibrational energy of the exciting gas with allowance for the gasdynamic losses during flow in a nozzle, the shock interaction of the individual jets, and the best conditions for stream mixing in the gasdynamic channel. Moreover, it is necessary to allow for possible technological problems in the fabrication of the nozzle unit.

The results of calculations of the gasdynamic parameters in the stream core for different configurations of the individual nozzles in the honeycomb unit are presented in Fig. 2. Although conical nozzles with a half-angle of 10° do provide a somewhat lower level of gasdynamic losses, as was also indicated in [13], one must keep in mind that an axisymmetric profiled nozzle with an angle of inclination of the generatrix at the cut also of 10° is two-thirds as long as a conical nozzle and provides a considerably higher rate of freezing-in in the acceleration section of the nozzle. This fact can have a decisive importance when combustion products containing admixtures of H_2 , CO , and CO_2 are used as the donor gas. The ratio of the stored energy of the exciting gas in the cut cross section of a nozzle to its value in the critical cross section for three compositions as a function of the binary similarity criterion $P_0 d_*$ is presented in Fig. 3. As in estimating the gasdynamic losses, the losses connected with the boundary layers in the nozzles and the interaction of streams at the cut

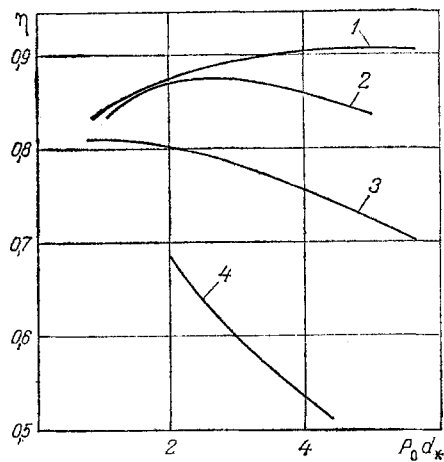


Fig. 3. Efficiency of an axisymmetric nozzle as a function of $P_0 d_*$ (10^{-3} MPa·m): $A/A_* = 36$; $T_0 = 2700^\circ\text{K}$; $\bar{T}_w = 0.3$; $\theta_a = 10^\circ$; 1) 100% N₂; 2) 90% N₂ + 10% H₂; 3) 35% N₂ + 55% CO + 10% H₂ (1-3: profiled nozzle); 4) 35% N₂ + 10% H₂ + 55% CO (conical).

are taken into account in the calculation. The appearance of hydrogen in the composition, especially together with CO, leads to considerable losses of vibrational energy in the nozzle, and in this case the advantage of profiled nozzles becomes obvious, especially when the content of these admixtures is high. The optimization of the geometrical characteristics of individual nozzles must be carried out for the specific composition of the exciting gas, and it is evidently impossible to draw conclusions in advance about the optimum value of $P_0 d_*$ for a nozzle of the exciting gas of a mixing GDL. Thus, for pure nitrogen the losses of vibrational energy are determined only by the presence of boundary layers and the shock interaction, and the nozzle parameters must be characterized by large $P_0 d_*$ and small angles at the cut. An increase in the H₂ and CO content leads to a decrease in the optimum values of the freezing-in parameter. It is interesting that at relatively low levels of hydrogen content the nozzle efficiency hardly depends on $P_0 d_*$, and in this case one must be oriented only toward providing the optimum mixing conditions when choosing the characteristic dimensions of the nozzle unit.

To conserve vibrational energy in the stream over a considerable distance in the resonator region (with allowance for the extended mixing zone) one must maintain a static temperature of $\sim 300^\circ\text{K}$, which leads to the need to increase the expansion ratio of the nozzles for the exciting gas with an increase in T_0 . For example, with allowance for viscous and shock losses it must be >80 at $T_0 \sim 2000^\circ\text{K}$. This requirement is stricter for mixtures with a considerable CO content, since with the lower rate of transfer of vibrational energy from CO molecules to CO₂ molecules than for nitrogen, we obtain acceptable values of the resonator efficiency its length along the stream must be ≥ 0.30 m [1]. On the other hand, when the expansion ratio of the nozzles for the exciting gas is increased while the expansion ratio of the nozzles for the radiating gas is fixed, the velocity expansion ratio (velocity ratio) of the mixing jets changes insignificantly. For example, when the Mach number is increased from 3 to 6 the velocity at the nozzle cut grows by about 10%, whereas the static temperature decreases twofold. Estimates of the level of freezing-in of vibrational energy during flow in the nozzle unit show that even for a relatively high content of admixtures one can assure a nozzle efficiency on the order of 70–80%.

A key problem for GDL with selective excitation is providing the maximum efficiency of transfer of vibrational energy during the mixing of the radiating and exciting gases in the gasdynamic channel. This process is characterized, on the one hand, by equalization of the stream parameters in the cross section of the gasdynamic channel and, on the other hand, by losses of vibrational energy which could be converted to radiation.

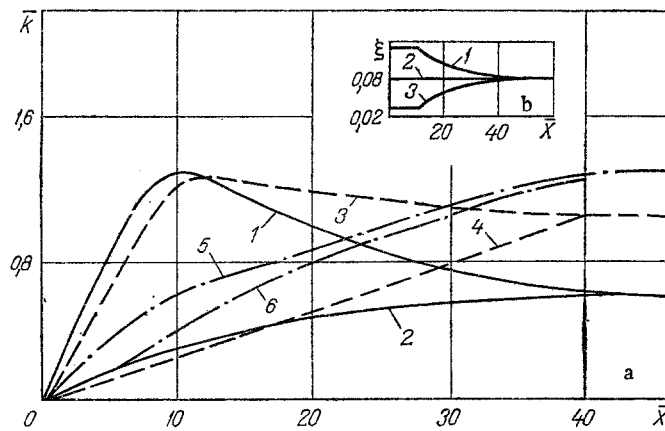


Fig. 4. Dependence of the average (2, 4, 6) and local (1, 3, 5) values of the amplification ratio (1/m) on the distance from the nozzle cut (a) for three laws of variation of the concentration in the mixing zone (b): 1, 2 (a): 1 (b); 3, 4 (a): 2 (b); 5, 6 (a): 3 (b); $T_{01} = 1000^\circ\text{K}$; $P_{01} = 0.5 \text{ MPa}$; $(A/A^*)_1 = 16$; $T_{02} = 2700^\circ\text{K}$; $P_{02} = 4.0 \text{ MPa}$; $(A/A^*)_2 = 36$.

The vibrational-nonequilibrium state in the mixing zone is determined to a considerable extent by the variation of the CO_2 concentration in it along the stream. Some possible variations of the CO_2 concentration for the same gasdynamic and vibrationally excited state of the mixing streams are shown in Fig. 4b. In the calculation the configuration of the mixing zone, and hence the assignment of the distribution of CO_2 concentration, is determined by the choice of the lengths L_1 and L_2 (see Fig. 1). The actual mechanism of turbulent mixing of axisymmetric jets is undoubtedly considerably more complicated and is determined by the gasdynamic and geometrical characteristics of the medium. The difference in the velocities of the mixing streams and the geometrical dimensions of the individual jets can be taken as the main factors, however [14]. It was shown long ago [13], in calculations based on the "instantaneous-mixing" model, that a considerable difference in the velocities of the exciting and radiating gases leads to an unacceptable increase in the losses of vibrational energy owing to the increase in the static temperature, so that in practice the only parameters through which one can alter the configuration of the mixing zone in one way or another evidently are the geometrical dimensions of the jets. An analysis of the influence of the mixing conditions on the efficiency of a mixing GDL shows that not only the length over which the complete mixing of the radiating and exciting gases occurs but also the rate of entrainment of one or the other component into the mixing zone has considerable importance for supersonic parallel jets of a vibrationally excited gas mixture. Rapid "washing out" of the core of the jet of exciting gas, characterized by an increased CO_2 concentration in the initial mixing section, leads to a considerable increase in the static temperature in the mixing zone and an increase in the losses of vibrational energy. The best conditions for conserving the vibrational energy stored in the gas are provided in the case when the mixing zone is formed through a gradual increase in the CO_2 concentration in it to the value calculated for complete mixing. A smooth increase in both the average and the local amplification ratios in the mixing section, as well as a higher level of amplification ratio after complete mixing, are characteristic of such conditions (Fig. 4a). These mixing conditions lead to considerable variation of the energy stored in the stream beyond the mixing zone. For variants 1, 2, and 3 (Fig. 4b) it is 70, 100, and 115 J/g, respectively.

Naturally, an analysis of complicated three-dimensional flows within the framework of a one-dimensional model cannot pretend to a high degree of accuracy and total reliability of the results obtained, but numerous examples of the successful application even of "instantaneous-mixing" models to such nozzle units indicate the promising nature of such an analysis of processes in mixing GDL, both from the point of view of the clearer definition of the separate factors and from the point of view of the clarification of their combined influence.

The results obtained indicate the possibility of the strong influence of the character of the mixing of the components, as well as of the composition of the exciting gas and the geometry of an individual nozzle, on the operating efficiency of a mixing GDL.

NOTATION

T, static temperature; u, velocity; P, pressure; μ , molecular weight; e_i , average number of vibrational quanta; k, amplification ratio for a weak signal; I, radiation intensity; ξ_i , molar fraction of the component in the mixture; δ^* , displacement thickness; r, d, current radius and diameter of the nozzle; θ , angle of inclination of the generating profile of the nozzle and characteristic temperature; h, enthalpy; G, flow rate; L, characteristic length; A/A*, expansion ratio of the nozzle; T_w , wall temperature; $\bar{T}_w = T_w/T_0$, temperature factor; $\eta = E_a^*/E_*$, nozzle efficiency; E, stored vibrational energy; $X = X/(r_{a1} + r_{a2})$, dimensionless coordinate. Indices: 1, radiating gas stream; 2, exciting gas stream; 0, stagnation value; $a, *, \sim$, values at the cut of the nozzle, in its critical cross section, and beyond the compression shock in the cross section of the nozzle cut.

LITERATURE CITED

1. F. E. Cassady, "Survey of advanced gasdynamic laser concepts," J. Energy, 4, No. 4, 145-161 (1980).
2. O. V. Achasov, R. I. Soloukhin, and N. A. Fomin, "A numerical analysis of the characteristics of a gasdynamic laser with selective thermal excitation and mixing in a supersonic stream," Kvantovaya Elektron. (Moscow), 5, No. 11, 2337-2341 (1978).
3. N. A. Fomin and S. M. Khizhnyak, "Peculiarities in the choice of mixing conditions in a CO₂-N₂-H₂O GDL," Inzh.-Fiz. Zh., 42, No. 1, 42-46 (1982).
4. P. V. Belkov, V. V. Val'ko, A. S. D'yakov, et al., "CO₂ GDL based on mixtures containing carbon monoxide," Kvantovaya Elektron. (Moscow), 7, No. 11, 2385-2392 (1980).
5. T. H. Yang, "Modeling of a continuous chemical laser with an annular unstable resonator," AIAA J., No. 10 (1980).
6. Chemical Lasers [Russian translation], Mir, Moscow (1980).
7. A. G. Velikanov, N. M. Gorshunov, Yu. A. Kunin, et al., "Optimization of the light amplification ratio in a supersonic stream with mixing of vibrationally excited N₂ with CO₂-He," Kvantovaya Elektron. (Moscow), 7, 1869-1875 (1980).
8. W. L. Shackelford, A. W. Witte, and J. E. Broadwell, "Experimental studies of chemical reactive (F + H₂) flow in supersonic free jet mixing laser," AIAA J., 12, 1009-1010 (1974).
9. A. S. Birykov, "Kinetics of physical processes in gasdynamic lasers," Tr. Fiz. Inst. Akad. Nauk SSSR, 83, 13-86 (1975).
10. V. S. Avduevskii and R. I. Kopyatkevich, "Calculation of the laminar boundary layer in a compressible gas for an arbitrary pressure distribution over a surface," Izv. Akad. Nauk SSSR, Mekh. Mashinostr., No. 1, 3-12 (1960).
11. L. H. Back and R. F. Cuffel, "Compressible laminar boundary layers with large acceleration and cooling," AIAA J., 14, No. 7, 968-971 (1976).
12. V. N. Uskov, "Interference of standing gasdynamic discontinuities," in: Supersonic Gas Jets [in Russian], Nauka, Novosibirsk (1982), pp. 22-46.
13. P. E. Cassady, J. F. Newton, and P. H. Rose, "A new mixing gasdynamic laser," AIAA J., 16, No. 4, 305-312 (1978).
14. Turbulent Mixing of Gas Jets [in Russian], Nauka, Moscow (1974).